ON THE EFFECTS OF THE ATTITUDES TOWARD RISK AND CORPORATE TAX ON THE FIRM'S INVESTMENT AND FINANCIAL DECISIONS

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Received on February 22, 1991, this submission was with the authors for two revisions and was accepted on May 12, 1993.

The objective of this paper is to study the investment and financial decisions of a firm taking into account the firm's attitude towards risk and corporate taxation. It is shown that the attitude towards risk affects the investment decision and not the financial decision. In particular, it is shown that a risk averse firm would invest less capital than a risk neutral firm though their leverages may not be different. It is also shown that both risk averse and risk neutral firms would have an optimal leverage when there is corporate tax as compared to the situation when there is no tax. It is further shown that

The Mid-Atlantic Journal of Business
Volume 30, Number 2, June 1994

there is a negative relation between the level of capital investment and leverage.

1. INTRODUCTION

A thirty years old contention in modern finance theory involves the issue of corporate financing policy and capital structure. Modigliani and Miller (1958) and their adherents such as Stiglitz (1969,1975), Hite (1977), Hellwig (1981) have shown that the market value of a firm is independent of its capital structure under certain assumptions which include the absence of taxes, transaction costs, and bankruptcy costs. As a result of this assertion, investment and financial decisions of a firm can be considered independent and hence can be made separately. On the other hand, Weston (1963), Bergles (1963) and Schwartz (1959) have shown that a value-maximizing, optimal capital structure does exist for a firm. Miller (1977) includes personal taxation in his model and shows that leverage irrelevance would hold for a single firm. This assertion has been challenged by DeAngelo and Masulis (1980), and Dammon and Senbet (1988).

The implication of leverage-relevance theory is that investment and financing decisions of a firm can not be considered as independent and hence must be made simultaneously. Interaction of these two decisions have been studied by a number of researchers.[1] In this paper we study the interaction of these two decisions, using the risk attitudes of a firm.

Sandmo (1971) and Hartman (1976) have studied the influence of the firm's attitude towards risk on the investment decisions of a firm. Mukherjee and Henderson (1987) have also shown that most firms explicitly consider risk in their analysis of capital investment. But nothing has been done to study the influence of risk attitudes of a firm on its financial decisions. This paper examines the effect of attitudes towards risk on the financial decisions of a firm.

An analytical model is developed in section 2. Section 3 uses the model to analyze the effects of risk attitude on the financial decisions of risk-neutral and risk-averse firms. Section 4 discusses the effects of corporate taxes on the investment behavior. Section 5 deals with the decision behavior under extreme cases, and section 6 offers conclusions.

2. ANALYTICAL MODEL

It is assumed that a competitive firm operates for a single period. At the beginning of the period, it invests capital (K), and produces an out-

[1]Recently, several researchers, such as Myers (1974), Hite (1977), Dotan & Ravid (1985), DeAngelo & Masulis (1980), and Dammon & Senbet (1988) have established their models based on this interaction. Furthermore, several researchers, such as Rendleman (1978) and Castanias (1983) have proved that the real and financial decisions are highly interrelated.

put (q) which is a function of the capital invested, or q = f(K), by the end of the period. At the end of the period, the firm would be liquidated and the net proceeds would be distributed to the stockholders.

The production costs (c) of output f(K) is assumed to be a twice-differentiable function c[f(k)] such that $c'[f(K)] > 0$ and $c''[f(K)] > 0$.

The uncertainty in the model is introduced in the form of a stochastic output price p with its distribution known ex ante, i.e.,[2] p = p(u), where u is a random variable satisfying:

$$\partial p/\partial u > 0$$

The firm will have to decide its investment (K) and debt (D) at the beginning of the period under this uncertainty. The firm is assumed to finance its investment by a combination of debt (D) and equity (S). The cost of debt is assumed to comprise a default risk premium which is positively related to the debt-equity ratio of the firm.[3] Under this assumption, cost function of debt is given by r(y) such that r(0) = r, and

$$r'(y) = 0 \quad \text{as} \quad 0 \leqq y \leqq \bar{y}$$
$$> 0 \quad \text{as} \quad y > \bar{y}$$

and

$$r''(y) = 0 \quad \text{as} \quad 0 \leqq y < \bar{y}$$
$$> 0 \quad \text{as} \quad y \geqq \bar{y}$$

Where: $y = D/S$ is the debt-equity ratio,
$\bar{y}$ = a fixed ratio, and
r = the market interest rate.[4]

This function asserts that the cost of debt remains fixed when leverage

[2]This uncertainty assumption is based on Leland's definition (1972), which is given by the following stochastic demand relationship.

$$q(p,q,u) = 0, \tag{A1}$$

where p and q are the output price and demand respectively, and u is a random variable. Implicit function (A1) can be solved for p, i.e. p = p(q, u), which can be reduced to p = p(u), in a competitive equilibrium market. Several researchers have adopted the similar definition of uncertainty, for example, Dotan & Ravid (1985) take the additive form such that p(u) = p + u.

[3]While this is an assumption in the model, this assumption is realistic. For example, the loan rates charged by banks usually increase as the debt-equity ratios of borrowers increase. In addition, the credit quality assigned by bond rating agencies such as Standard & Poor's and Moody's usually declines as debt-equity ratio increases. Since the credit quality is negatively related to the bond issue's risk premium, a positive relationship between risk premium and debt-equity ratio is established.

[4]We acknowledge that this assumption has implication for the results obtained in the paper. This assumption, however, is widely used either explicitly or implicitly in the literature. These papers that have also suggested that yields demanded by lenders are positively related to the debt-equity ratio of the borrower include Modigliani & Miller (1958), Steigum (1983), Koutsoyiannis (1982), Hochman et al (1973), Alberts & Hit (1983), and Kim (1978).

is at a low level, but increases when leverage is greater than or equal to a fixed $\bar{y}$.

The firm is assumed to maximize the expected utility of net equity wealth, rather than maximizing the market value of the firm.[5] The firm's utility function is described by a Von Neumann-Morgenstern utility function. Let U stand for utility and π for net equity wealth. Then the firm's utility is given by $U = U(\pi)$; where $U'(\pi) > 0$, and $U''(\pi) < 0$ or $U''(\pi) = 0$ depending on whether the firm is risk-averse or risk-neutral.[6]

It is also assumed that there is corporate taxation and that interest payments are full tax deductible.

In this model, the problem that is faced by the firm is to determine both its investment level (K) and its debt level (D) simultaneously in order to maximize the expected utility of net equity wealth,[7] where net equity wealth is defined as the end-of-period cash flows to equity holders deducting the future value of the initial equity investment invested at the risk free rate (r), i.e.

$$\pi = p(u)f(K) - c(f(K)) - \tau[p(u)f(K) - c(f(K)) - r(y)D - \delta K] - D[1 + r(y)] + (1 - \delta)K - (1 + r)(K - D), \quad (1)$$

where τ is the corporate tax rate, and δ is the depreciation rate.

The first term is the revenue. The second term denotes the cost of production, and the third gives the taxes to be paid. The fourth term denotes the claim of the debt holders. The fifth is the salvage value of the firm, and the sixth term provides the cash flow to equity holders if their investment is invested at the risk free rate.

Since $y = D/S$ and $D = K - S$, we can consider K and S as the decision variables (instead of K and D). Once K and S are decided upon, D can be determined as $K - S$. Then expression (1) can be rewritten as

[5]Many researchers, such as King (1974), Alberts & Hite (1983), and Rendleman (1978) have taken this assumption into account in their analytic models. Mukherjee & Henderson also assert, in their survey paper (1987), that it is generally assumed that management's primary goal is to maximize the wealth of the firm's shareholders.

[6]For the detailed explanation, see Hey (1981), pp. 20–27.

[7]We can also take account of the more general case that the firm can decide the levels of capital (K) and labor (L), and hence the quantity of output (q), which are related by the production function, $q = Q(K,L)$, at the beginning of the period. Under the assumption that the goal of a firm is to maximize its expectation of the utility of net equity wealth, it can be found that a firm uses the least cost combination of K and L to produce the quantity of output it chooses. The detailed discussion can be found in Tseng and Yang (1987) or Holthausen (1976). Thus, the firm can decide its input levels along the curve consisting of points (K,L) satisfying the least cost combination. This curve can determine L as a function of K, i.e. $L = L(K)$, and therefore $q = Q(K,L(K)) = f(K)$ (see Tseng & Yang (1987)). Thus, the results in this paper also hold in the general case on the basis of the above discussion.

$$\pi = [p(u)f(K) - c(f(K)) - \delta K](1-\tau) + \tau r\left(\frac{K-S}{S}\right)\cdot(K-S) - r\left(\frac{K-S}{S}\right)(K-S) - rS \tag{2}$$

With this definition of net equity wealth, the problem for the firm can be written as

$$\underset{K,S}{\text{Max}}\; E[U(\pi)]. \tag{3}$$

3. THE ANALYSIS OF THE FIRM'S DECISIONS

In a previous study the authors have shown that, in the absence of corporate taxes, the firm's optimal leverage will be any low level (y) such that $y < \bar{y}$ whether the firm is risk-averse or risk-neutral. They have also shown that a risk-averse firm will invest less capital than a risk-neutral firm, other things being equal. Here we take into account the corporate taxation and analyze the investment and financial decisions.

The first-order conditions are given by

$$\frac{\partial E}{\partial K} = E\left\{U'(\pi)\left[(p(u)f'(K) - c'(f(K))f'(K) - \delta)(1-\tau) + \tau r'\left(\frac{K-S}{S}\right)\cdot\frac{K-S}{S} + \tau r\left(\frac{K-S}{S}\right) - r'\left(\frac{K-S}{S}\right)\cdot\frac{K-S}{S} - r\left(\frac{K-S}{S}\right)\right]\right\} = 0 \tag{4}$$

$$\frac{\partial E}{\partial S} = E\left\{U'(\pi)\left[-\tau r'\left(\frac{K-S}{S}\right)\frac{K(K-S)}{S^2} - \tau r\left(\frac{K-S}{S}\right) + r'\left(\frac{K-S}{S}\right)\frac{K(K-S)}{S^2} + r\left(\frac{K-S}{S}\right) - r\right]\right\} = 0 \tag{5}$$

The derivation of second-order conditions is shown in Appendix 1. Since the second-order conditions hold, the optimal capital and equity levels can be uniquely determined by the sufficient and necessary conditions (4) and (5). We denote the optimal capital and equity levels for a risk-averse firm by K_a and S_a, and for a risk-neutral firm by K_n and S_n.

For a risk-neutral firm, equation (4) can be rewritten as

$$E\{[p(u)f'(K) - c'(f(K))f'(K) - \delta](1-\tau)\} + \tau r'\left(\frac{K-S}{S}\right)\cdot\frac{K-S}{S} + \tau r\left(\frac{K-S}{S}\right) = r'\left(\frac{K-S}{S}\right)\cdot\frac{K-S}{S} + r\left(\frac{K-S}{S}\right), \tag{6}$$

since $U'(\pi)$ is constant. This means that the expectation of the sum of the marginal revenue and the marginal tax shelter induced by increasing capital investment by one unit is equal to the marginal debt cost that results from a one unit increase in capital investment.

For the risk-averse firm, it can be shown that

$$E\{[p(u)f'(K) - c'(f(K))f'(K) - \delta](1 - \tau)\} + \tau r'\left(\frac{K-S}{S}\right)\cdot\frac{K-S}{S} + \tau r\left(\frac{K-S}{S}\right) > r'\left(\frac{K-S}{S}\right)\cdot\frac{K-S}{S} + r\left(\frac{K-S}{S}\right). \quad (7)$$

(see Appendix 2).

That is, the risk-averse firm requires that the expectation of the sum of the marginal revenue and the marginal tax shelter be larger than the marginal debt cost.

We let

$$A = -\tau r'\left(\frac{K-S}{S}\right)\frac{K(K-S)}{S^2} - \tau r\left(\frac{K-S}{S}\right) + r'\left(\frac{K-S}{S}\right)\frac{K(K-S)}{S^2} + r\left(\frac{K-S}{S}\right) - r. \quad (8)$$

Since A is independent of u, equation (5) can be reduced to

$$r\left(\frac{K-S}{S}\right) + r'\left(\frac{K-S}{S}\right)\cdot\frac{K(K-S)}{S^2} = \frac{r}{1-\tau} \quad (9)$$

which holds for both risk-averse and risk-neutral firms. We denote the optimal leverages of a risk-averse firm and of a risk-neutral firm by y_a and y_n respectively, which are given by

$$y_a = \frac{K_a - S_a}{S_a} \quad \text{and} \quad y_n = \frac{K_n - S_n}{S_n}$$

Through the properties of debt cost function r(y) and equation (9) we can obtain that

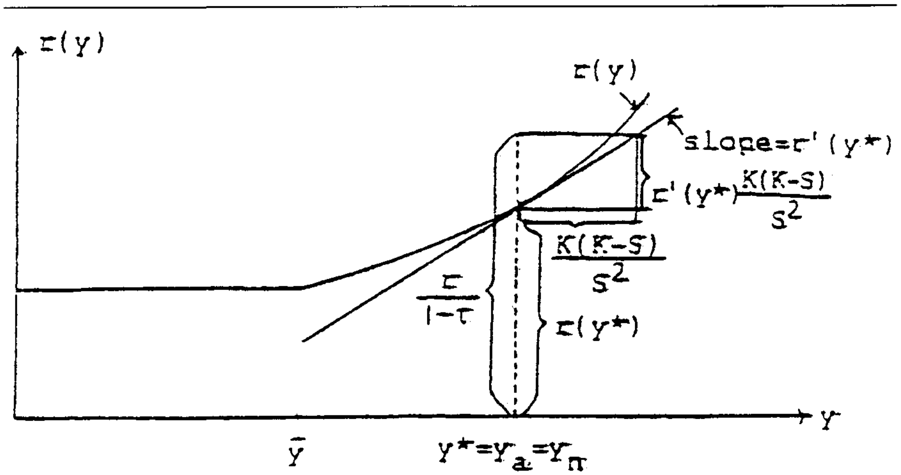
$$\bar{y} < y_a = y_n \quad (10)$$

(see Appendix 2).

This conclusion can be expressed graphically as shown in Figure 1.

In the absence of corporate tax, it was shown that a firm would select any leverage y such that $y < \bar{y}$, but in the case with tax, the optimal leverage y^* exists that satisfies $y^* > \bar{y}$. Thus the introduction of corporate tax would cause a firm to raise its leverage.

FIGURE 1



In the absence of corporate tax, both risk-averse and risk-neutral firms would have low leverage. Now, from the equality in (10), we can say that their leverages can be the same even with corporate taxation. Thus a risk-averse and a risk-neutral firms may have the same leverage whether corporate tax exists or not. In the absence of corporate tax, a risk-averse firm will invest less capital than the risk-neutral firm under the same production conditions. This assertion also holds in the case with corporate tax, since we can obtain,

$$K_a < K_n \qquad \text{and} \qquad S_a < S_n \tag{11}$$

Therefore, a risk-averse firm's investment level will be less than that invested by a risk-neutral firm under the same production conditions, whether there is corporate tax or not. The derivation of inequality (11) is presented in Appendix 2.

We denote equations (4) and (5) by $E_k = 0$ and $E_s = 0$ respectively. By using the second-order conditions (see Appendix 1), we can obtain

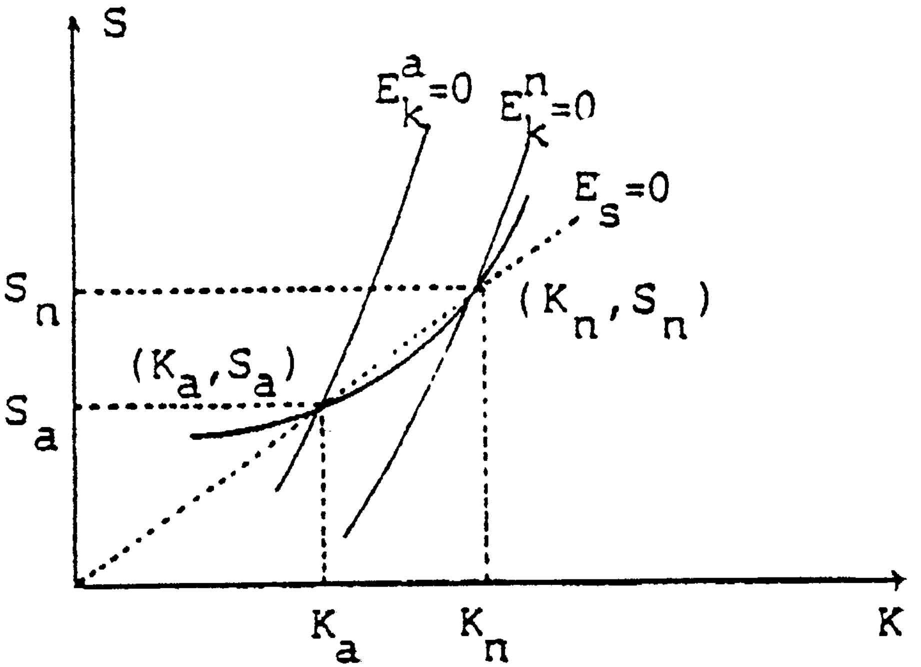
$$\left.\frac{dS}{dK}\right|_{E_k=0} = -\frac{E_{kk}}{E_{ks}} > 0,$$

$$\left.\frac{dS}{dK}\right|_{E_s=0} = -\frac{E_{ks}}{E_{ss}} > 0,$$

and

$$\left.\frac{dS}{dK}\right|_{E_k=0} - \left.\frac{dS}{dK}\right|_{E_s=0} = \frac{E_{ks}^2 - E_{kk}\,E_{ss}}{E_{ks}\,E_{ss}} > 0$$

FIGURE 2



That is, the tangent rate of the curve $E_k = 0$ is larger than the tangent rate of the curve $E_s = 0$ at the solution points which determine the firm's optimal decisions. We also denote the equation (4) for a risk-averse firm and for a risk-neutral firm by $E_k^a = 0$ and $E_k^n = 0$ respectively. From the inequalities in (11), it can be obtained that the curve $E_k^a = 0$ is at the left-hand side of the curve $E_k^n = 0$. The above discussion is represented in Figure 2.

From equality (10), we can obtain that the points (K_a, S_a) and (K_n, S_n) are on the line which goes through the origin.

This discussion leads to Proposition 1 as follows:

Proposition 1

A risk-neutral firm will determine its capital and equity levels by equating the expectation of the sum of the marginal revenue and the marginal tax shelter to the marginal debt costs. A risk-averse firm will determine its capital and equity levels such that the sum of the marginal revenue and the marginal tax shelter would be larger than the marginal debt costs. In general, a risk-averse firm will invest less capital and equity than the risk-neutral firm, although their leverages may not be different.

4. THE EFFECTS OF CORPORATE TAX ON THE INVESTMENT DECISIONS

We have shown that the introduction of corporate tax would cause a firm to raise its leverage and to select an optimal leverage. Furthermore, the introduction of corporate tax would also cause a firm to reduce its capital investment. We denote the optimal capital for a risk-averse firm by K_a^a in the absence of corporate tax and by K_a^p in the presence of corporate tax, and denote the optimal capital for a risk-neutral firm by K_n^a (in the absence of corporate tax) or K_n^p (in the presence of corporate tax).

It can be shown that

$$K_a^p < K_a^a \qquad \text{and} \qquad K_n^p < K_n^a \tag{12}$$

The derivation is presented in Appendix 3.

Consequently, the absence of tax will encourage a firm to raise its investment level. In the presence of corporate tax, capital investment will decline, though the firm will increase its leverage resulting from the tax deductibility of interest payments. In general, other things being equal, the capital level is inversely related to the use of leverage, since

$$\frac{dK_a}{dy^*} < 0, \qquad \text{and} \qquad \frac{dK_n}{dy^*} < 0, \tag{13}$$

which are derived in Appendix 4, where $y^* = y_a = y_n$.

These results are similar to that derived by Dotan and Ravid (1985), but contrast with Hite's analogous theorem (1977). In Hite's model, a firm can utilize debt-related tax shields all the time, so increasing debt lowers the user cost of capital and causes the capital investment to increase. In Dotan & Ravid's model, an increase in debt leads to a decrease in expected tax shelter of capital by increasing the probability of accounting loss, and hence reduces the capital investment. In this paper, an increase in leverage will cause the debt cost to sharply increase and to be larger than the tax shelter. Consequently, increasing leverage will lower the capital level.

We turn now to an investigation of the effects of a change in the tax rate on the firm's decision. We denote the left-hand side in equation (9) by $h(y)$, where $y = (K - S)/S$, i.e.

$$h(y) = r(y) + r'(y) \cdot (y + y^2),$$

which is evaluated at the optimal leverage y^*. We can obtain that

$$\frac{dh}{dy} \cdot \frac{dy}{d\tau} = \frac{r}{(1-\tau)^2} > 0$$

and

$$\frac{dy}{d\tau} > 0. \tag{14}$$

Through inequalities (13) and (14), it can be shown that

$$\frac{dK_a}{d\tau} < 0 \quad \text{and} \quad \frac{dK_n}{d\tau} < 0.$$

Therefore, it can be concluded that the firm's leverage will rise, whereas capital levels will decline independent of their attitudes toward risk, as long as the tax rate increases.

These conclusions are similar to those derived by Dotan & Ravid, although the firm's attitude toward risk has been considered in our analytic model. This leads to Proposition 2:

Proposition 2

Other things being equal, the introduction of corporate tax will reduce the firm's capital investment, but will raise the firm's leverage, whether the firm is risk-averse or risk-neutral. Therefore, there exists a negative relation between capital level and leverage. Furthermore, the capital level will decrease and the leverage will increase as the tax rate rises.

5. DECISION BEHAVIOR UNDER THE EXTREME CASE OF THE COST FUNCTION OF DEBT

In section 2, we asserted that a firm will choose low leverage (y) such that $y \leqq \bar{y}$ in the absence of corporate tax. Furthermore, in the extreme case of the cost function of debt such that $\bar{y} \to \infty$, in which there is no bankruptcy possibility, a firm can select any leverage which does not affect the expectation of utility of its net equity or the expectation of its net equity. This assertion completely confirms the Modigliani and Miller leverage neutrality theory.

In addition to the introduction of corporate tax, we take into account the extreme case of the cost function of debt. In this situation, the firm's optimal leverage would not exist, whereas it exists in the general case. In this extreme case, we have $r(y) = r$ and $r'(y) = 0$ for all $y \geqq 0$, hence there is no solution for equation (9). In analytic model (3), the representation of π can be revised as

$$\pi = [p(u)f(K) - c(f(K)) - \delta K](1-\tau) + \tau rD - rK.$$

This representation reflects that at any fixed capital level (K), the more debt a firm has, the more net equity wealth it obtains, and hence the more utility it has. This assertion supports the Modigliani and Miller suggestion that a firm can maintain more debt financing with the introduction of corporate tax (Modigliani & Miller, 1958). In reality, an increase in debt would increase the financial risk, and hence causes the cost of debt to rise. Furthermore, the increase in debt cost would more than offset the advantages due to the deductibility of interest payments, and would induce a firm to select an appropriate leverage.

6. CONCLUSIONS

In this paper, we investigate the firm's decision behavior by taking into account the firm's attitude toward risk in the presence of corporate tax, under the interaction of investment and financial decisions. In general, the investment decision will be affected by the attitude toward risk, but the financial decision will not. However, the capital level is inversely related to the use of leverage. Other things being equal, a risk-averse firm will invest less capital than a risk-neutral firm, whether there is corporate tax or not, but their leverages may not be different (Proposition 1). One implication of Proposition 1 is that a government can induce firms' capital investments by reducing the uncertainty or risks faced by firms.

The imposition of corporate tax causes a firm to choose an optimal leverage, while the leverage may be any low level in the absence of tax. An increase in tax rate will lead a firm to raise its leverage, and reduce its capital investment, whether the firm is risk-averse or risk-neutral (Proposition 2). Some of these results are similar to those derived by Dotan & Ravid. The results in Proposition 2 imply that the reduction of corporate tax rate may provide an incentive for firms to increase their capital investments.

Though we can confirm the Modigliani and Miller theorems concerning financial decisions in the extreme case of cost function of debt such that $\bar{y} \rightarrow \infty$, we suggest that a firm can choose any low leverage in the absence of tax, but select an optimal leverage in the presence of corporate tax. In any event, the firm's decisions will be affected by its attitude toward risk, since most firms indeed take into account the risk in their analysis of capital investments.

APPENDIX 1

Appendix 1. The derivation of the second-order conditions for analytic model (3).

From equations (4) and (5), we have

$$E_{kk} = \frac{\partial^2 E}{\partial K^2} = E\left\{U''(\pi)\left[(p(u)f'(K) - c'(f(K))f'(K) - \delta)(1-\tau) - r'\left(\frac{K-S}{S}\right)\left(\frac{K-S}{S}\right)(1-\tau) - r\left(\frac{K-S}{S}\right)(1-\tau)\right]^2 - U'(\pi)\left(\left[p(u)f''(K) - \frac{d^2}{dK^2}c(f(K))\right](1-\tau) + F\right)\right\}$$

$$E_{sk} = E_{ks} = \frac{\partial^2 E}{\partial S \partial K} = E\left\{U'(\pi)\frac{K}{S}F\right\}$$

$$E_{ss} = \frac{\partial^2 E}{\partial S^2} = E\left\{U''(\pi)\left[r'\left(\frac{K-S}{S}\right)\left(\frac{K(K-S)}{S^2}\right)(1-\tau) + r\left(\frac{K-S}{S}\right)(1-\tau) - r\right]^2 - U'(\pi)\frac{K^2}{S^2}F\right\},$$

which are evaluated at the solution point of equations (4) and (5), where

$$F = r''\left(\frac{K-S}{S}\right)\left(\frac{K-S}{S^2}\right)(1-\tau) + \left(\frac{2}{S}\right)r'\left(\frac{K-S}{S}\right)(1-\tau).$$

The Hessian function is given by

$$|H| = \begin{vmatrix} \dfrac{\partial^2 E}{\partial K^2} & \dfrac{\partial^2 E}{\partial S \partial K} \\ \dfrac{\partial^2 E}{\partial K \partial S} & \dfrac{\partial^2 E}{\partial S^2} \end{vmatrix}$$

$$= \frac{\partial^2 E}{\partial K^2} E\left\{U''(\pi)\left[r'\left(\frac{K-S}{S}\right)\left(\frac{K(K-S)}{S^2}\right)(1-\tau) + r\left(\frac{K-S}{S}\right)(1-\tau) - r\right]^2\right\} + E\left\{U''(\pi)\left[(p(u)f'(K) - c'(f(K))f'(K) - \delta)(1-\tau) - r'\left(\frac{K-S}{S}\right)\left(\frac{K-S}{S}\right)(1-\tau) - r\left(\frac{K-S}{S}\right)(1-\tau)\right]^2 - U'(\pi)\left(\left[p(u)f''(K) - \frac{d^2}{dK^2}c(f(K))\right](1-\tau)\right)\right\} \cdot E\left\{-U'(\pi)\frac{K^2}{S^2}F\right\}$$

Since $f''(K) < 0$, $F > 0$, $d^2[c(f(K))]/dK^2 > 0$, and $U''(\pi) < 0$, $U'(\pi) > 0$ for all u, we can obtain that

$$E_{kk} < 0,\ E_{sk} = E_{ks} > 0 \qquad \text{and} \qquad E_{ss} < 0$$

and hence $|H| > 0$.

Therefore, the second-order conditions hold for the risk-averse firm. Since $U''(\pi) = 0$ for the risk-neutral firm, it is more easier to show that the second-order conditions also hold for the risk-neutral firm.

APPENDIX 2

Appendix 2. The derivations of (7), (10) and (11).

For the risk-neutral firm, equation (4) can be rewritten by

$$\frac{\partial E}{\partial K} = E\left\{[[p(u) - c'(f(K_n))]\,f'(K_n) - \delta]\,(1-\tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right)\cdot\frac{K_n - S_n}{S_n} + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right)\right\} = 0. \qquad \text{(A2.1)}$$

Since $\partial p/\partial u > 0$ and $f'(K) > 0$, $\partial E/\partial K$ is a increasing function of u. By using the mean value theorem for integrals, there exists a u-value, say u_n, such that

$$\{[p(u_n) - c'(f(K_n))]f'(K_n) - \delta\}(1-\tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right) = 0$$

and

$$\begin{cases} \{[p(u) - c'(f(K_n))]f'(K_n) - \delta\}(1-\tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} \\ \quad + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right) > 0, \quad \text{as } u > u_n \\ \{[p(u) - c'(f(K_n))]f'(K_n) - \delta\}(1-\tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right)\left(\frac{K_n - S_n}{S_n}\right) \\ \quad + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right) < 0, \quad \text{as } u < u_n \end{cases} \qquad \text{(A2.2)}$$

From (2), we have $\partial\pi/\partial u > 0$, which implies that

$$\begin{cases} \pi(u) > \pi(u_n), & \text{as } u > u_n \\ \pi(u) < (u_n), & \text{as } u < u_n \end{cases}, \tag{A2.3}$$

for any capital level K. Using $U''(\pi) < 0$ for the risk-averse firm and inequalities (A2.3), we obtain

$$\begin{cases} U'(\pi) < U'(\bar{\pi}), & \text{as } u > u_n \\ U'(\pi) > U'(\bar{\pi}), & \text{as } u < u_n \end{cases}$$

where $\bar{\pi} = \pi(u_n)$.

Through (A2.2) and (A2.4), it can be shown that

$$U'(\pi)\left\{[[p(u) - c'(f(K_n))]\, f'(K_n) - \delta](1 - \tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right) \cdot \frac{K_n - S_n}{S_n} + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right)\right\} <$$

$$U'(\bar{\pi})\left\{[[p(u)\, c'(f(K_n))]f(K_n) - \delta](1 - \tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right)\right\}$$

hold for all *u*. Therefore, we can obtain

$$E\left\{U'(\pi)[[p(u) - c'(f(K_n))]\, f'(K_n) - \delta](1 - \tau) + \tau r'\left(\frac{K_n - S_n}{S_n}\right) \cdot \frac{K_n - S_n}{S_n} + \tau r\left(\frac{K_n - S_n}{S_n}\right) - r'\left(\frac{K_n - S_n}{S_n}\right)\frac{K_n - S_n}{S_n} + r\left(\frac{K_n - S_n}{S_n}\right)]\right\} < 0 \tag{A2.5}$$

Equation (9) can be revised by

$$h(y) = r(y) + r'(y) \cdot (1 + y)y = \frac{r}{1 - \tau}. \tag{A2.6}$$

Since $h'(y) > 0$, a unique solution of equation (A2.6) exists, say y^*. In this case, we have

$$y_a = y_n = y^*$$

because equation (9), and hence equation (A2.6), hold for both the risk-averse firm and the risk-neutral firm. Since $\bar{y}$ satisfies

$$r(\bar{y}) = r \qquad \text{and} \qquad r'(\bar{y}) = 0,$$

it must be that

$$\bar{y} < y_a = y_n.$$

Therefore, (10) has been proved. The above equality can imply that

$$\frac{K_a}{S_a} = \frac{K_n}{S_n}$$

We let $\alpha = S_a/K_a$ and maximize $E[U(\pi)]$ along the line $\alpha K - S = 0$, so model (3) can be reduced to

$$\max_K E[U(\pi)] \tag{A2.7}$$

where

$$\pi = [p(u)f(K) - c(f(K)) - \delta K](1 - \tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha)K - r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha)K - \alpha rK.$$

The first-order condition is given by

$$\frac{dE}{dK} = E\left\{U'(\pi)\left[[p(u)f'(K) - c'(f(K))f'(K) - \delta](1-\tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - \alpha r\right]\right\} = 0.$$

The second-order conditions of model (3), which are derived in Appendix 1, guarantee the second-order condition of model (A2.7). The solutions of model (3), (K_a,S_a) and (K_n,S_n), are also the solutions of model (A2.7) for the risk-averse firm and for the risk-neutral firm respectively. Thus, we have

$$\left.\frac{dE}{dK}\right|_{K=K_a} = E\left\{U'(\pi)\left[[p(u)f'(K_a) - c'(f(K_a))f'(K_a) - \delta](1-\tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - r\left(\frac{1-\alpha}{\alpha}\right)(1-\alpha) - \alpha r\right]\right\} = 0. \tag{A2.8}$$

and

$$\left.\frac{dE}{dK}\right|_{K=K_n} = E\left\{[p(u)f'(K_n) - c'(f(K_n))f(K_n) - \delta](1-\tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - \alpha r\right\} = 0 \tag{A2.9}$$

Using a similar way for deriving inequality (A2.5), we can show that

$$E\left\{U'(\pi)[[p(u)f'(K_n) - c'(f(K_n))f'(K_n) - \delta](1-\tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - \alpha r\right\} < 0 \quad \text{(A2.10)}$$

The second-order condition of model (A2.7) guarantee that dE/dK is a decreasing function of K. Therefore, equality (A2.8) and inequality (A2.10) imply that

$$k_a < k_n \quad \text{(A2.11)}$$

Similarly, for the risk-neutral firm, the second-order condition implies that

$$E\left\{[p(u)f'(K) - c'(f(K))f'(K) - \delta](1-\tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - \alpha r\right\}$$

is a decreasing function of K, and hence, through equality (A2.9) and inequality (A2.11), we have

$$E\left\{[p(u)f'(K_a) - c'(f(K_a))f'(K_a) - \delta](1-\tau) + \tau r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - r\left(\frac{1-\alpha}{\alpha}\right)\cdot(1-\alpha) - \alpha r\right\} > 0.$$

Substituting (9) into above inequality, we can show that the following relationship holds:

$$E\{[p(u)f'(K_a) - c'(f(K_a))f'(K_a) - \delta](1-\tau)\} + \tau r'\left(\frac{K_a - S_a}{S_a}\right)\cdot\frac{K_a - S_a}{S_a} + \tau r\left(\frac{K_a - S_a}{S_a}\right) > r'\left(\frac{K_a - S_a}{S_a}\right)\frac{K_a - S_a}{S_a} + r\left(\frac{K_a - S_a}{S_a}\right)$$

Therefore, inequality (7) has been proved. Since $K_a/S_a = K_n/S_n$, inequality (A2.11) implies that the inequalities in (11) are satisfied.

APPENDIX 3

Appendix 3. The derivation of (12).

In the case of no corporate tax, the analytic model is given by

$$\max_{K,S} E[U(\pi)] \tag{A3.1}$$

where

$$\pi = p(u)f(K) - c(f(K)) - \delta K - r\left(\frac{K-S}{S}\right)(K-S) - rS$$

The first-order conditions are given by

$$\frac{\partial E}{\partial K} = E\left\{U'(\pi)\left[p(u)f'(K) - c'(f(K))f'(K) - \delta - r'\left(\frac{K-S}{S}\right)\left(\frac{K-S}{S}\right) - r\left(\frac{K-S}{S}\right)\right]\right\} = 0 \tag{A3.2}$$

and

$$\frac{\partial E}{\partial S} = E\left\{U'(\pi)\left[r'\left(\frac{K-S}{S}\right)\left(\frac{K(K-S)}{S^2}\right) + r\left(\frac{K-S}{S}\right) - r\right]\right\} = 0.$$

By the properties of $r(y)$, it can be obtained that the optimal leverage can be any low leverage y such the $y \leqq \bar{y}$ (see Tseng and Yang (1987)). At this situation, we have $r'(y) = 0$, and equation (A3.2) can be reduced to

$$E\{U'(\pi)[p(u)f'(K_a^a) - c'(f(K_a^a))f'(K_a^a) - \delta - r]\} = 0. \tag{A3.3}$$

Equation (4) can be reduced to

$$E\left\{U'(\pi)\left[p(u)f'(K_a^p) - c'(f(K_a^p))f'(K_a^p) - \delta - r'\left(\frac{K_a^p - S_a}{S_a}\right)\frac{K_a^p - S_a}{S_a} - r\left(\frac{K_a^p - S_a}{S_a}\right)\right]\right\} = 0$$

Since $u'(\pi) < 0$ for all u, $r'((K_a^p - S_a)/S_a) > 0$ and $r((K_a^p - S_a)/S_a) > r$, we can obtain that

$$E\{U'(\pi)[p(u)f'(K_a^p) - c'(f(K_a^p))f'(K_a^p) - \delta - r]\} > 0 \tag{A3.4}$$

We now consider the case with no debt and no corporate tax.

The analytic model is given by

$$\max_{K} E[U(\pi)]$$

where

$$\pi = p(u)f(K) - c(f(K)) - (r + \delta)K.$$

The first-order condition is given by

$$\frac{dE}{dK} = E\{U'(\pi)\,[p(u)f'(K) - c'(f(K))f'(K) - (r + \delta)]\} = 0$$

The second-order condition implies that dE/dK is a decreasing function of K. Therefore, by equality (A3.3) and inequality (A3.4), it can be obtained that

$$K_a^p < K_a^a.$$

Similarly, we can obtain that

$$K_n^p < K_n^a$$

APPENDIX 4

Appendix 4. The derivation of (13).

Since $y^* = (K_a - S_a)/S_a = (K_n - S_n)/S_n$, equation (4) can be written as

$$\frac{\partial E}{\partial K}(K_a, y^*) = 0$$

Through the total differentiation of the above equation with respect to K_a and y^*, we have

$$\frac{dK_a}{dy^*} = -\frac{\partial^2 E}{\partial y^* \partial K} \Big/ \frac{\partial^2 E}{\partial K^2} \qquad \text{(A4.1)}$$

where the value is evaluated at (K_a, y^*). From equation (4), we have

$$\frac{\partial^2 E}{\partial y^* \partial K} = -(1 - \tau)\,[r''(y^*) \cdot y^* + 2r'(y^*)] < 0 \qquad \text{(A4.2)}$$

Since $\partial^2 E/\partial K^2 < 0$, (A4.1) and (A.4.2) imply that

$$\frac{dK_a}{dy^*} < 0$$

Similarly, we can show that $dK_n/dy^* < 0$

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